

LESSON 4.3 Skills Practice

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The Factors of Life
The Factor Theorem and Remainder Theorem

Vocabulary

Remainder Theorem	Factor Theorem
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Choose the term from the box that best completes each statement.

- The _____ states that a linear polynomial $(x - r)$ is a factor of a polynomial $p(x)$ if and only if $p(r) = 0$ and $\frac{p(x)}{(x - r)}$ has a remainder of zero.
- The _____ states that when any polynomial equation or function is divided by a linear factor $(x - r)$, the remainder is the value of the equation or function when $x = r$.

Problem Set



Determine each function value using the Remainder Theorem. Explain your reasoning.

- Determine $p(3)$ if $p(x) = 2x^3 - 6x^2 - 36x - 36$.

$$\begin{array}{r}
 2x^3 + 0x^2 - 36 \\
 x - 3 \overline{) 2x^3 - 6x^2 - 36x - 36} \\
 \underline{2x^3 - 6x^2} \\
 -36x - 36 \\
 \underline{-36x + 108} \\
 -144
 \end{array}$$

When $p(x)$ is divided by $x - 3$, the remainder is -144 . So, by the Remainder Theorem $p(3) = -144$.

- Determine $p(-2)$ if $p(x) = x^4 - 10x^3 + 8x^2 + 106x - 105$.

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3. Determine $p(-3)$ if $p(x) = 2x^4 + 5x^3 + 8x^2 + 15x + 6$.

4. Determine $p(1)$ if $p(x) = x^4 + 3x^3 - 6x^2 - 8x$.

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5. Determine $p(10)$ if $p(x) = 6x^3 + 11x^2 - 3x - 2$.

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6. Determine $p\left(\frac{1}{3}\right)$ if $p(x) = x^4 - x^3 + 7x^2 - 9x - 18$.

Use the Factor Theorem to determine whether the given expression is a factor of each polynomial. Explain your reasoning.

7. Is $x - 2$ a factor of $f(x) = x^3 + 8x^2 - 31x + 22$?

If $x - 2$ is a factor of $f(x)$, then by the Factor Theorem $f(2) = 0$.

$$f(2) = (2)^3 + 8(2)^2 - 31(2) + 22$$

$$f(2) = 8 + 32 - 62 + 22$$

$$f(2) = 0$$

When $f(x)$ is evaluated at 2, the result is 0. According to the Factor Theorem $x - 2$ is a factor of $f(x)$.

8. Is $x - 3$ a factor of $f(x) = 4x^4 - x^3 - 52x^2 - 35x + 12$?

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9. Is $x - 12$ a factor of $f(x) = x^4 - 12x^3 + x^2 - 12x$?

10. Is $x - 8$ a factor of $f(x) = x^3 - 7x^2 - 14x + 48$?

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11. Is $x - 5$ a factor of $f(x) = x^3 + 5x^2 - x - 5$?

12. Is $3x + 4$ a factor of $f(x) = 3x^3 + 13x^2 + 18x + 8$?

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Use the Factor Theorem to determine whether $g(x)$ is the factored form of $f(x)$. Explain your reasoning.

13. Is
- $g(x) = (x + 8)(x - 1)(x + 2)$
- the factored form of
- $f(x) = x^3 - 7x^2 - 10x + 16$
- ?

$$f(-8) = (-8)^3 - 7(-8)^2 - 10(-8) + 16$$

$$f(-8) = 512 - 448 + 80 + 16$$

$$f(-8) = 160$$

$$f(1) = (1)^3 - 7(1)^2 - 10(1) + 16$$

$$f(1) = 1 - 7 - 10 + 16$$

$$f(1) = 0$$

$$f(-2) = (-2)^3 - 7(-2)^2 - 10(-2) + 16$$

$$f(-2) = -8 - 28 + 20 + 16$$

$$f(-2) = 0$$

No, the function $g(x)$ is not the factored form of $f(x)$. Since $f(-8) = 160$, $x + 8$ is not a factor of $f(x)$ by the Factor Theorem.

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14. Is
- $g(x) = (x - 3)(x + 5)(x + 2)(x - 1)$
- the factored form of
- $f(x) = x^4 + 3x^3 - 15x^2 - 19x + 30$
- ?

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15. Is $g(x) = (x - 2)(x + 9)(x + 1)$ the factored form of $f(x) = x^3 + 8x^2 - 11x - 18$?

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16. Is $g(x) = x(x - 4)(x - i\sqrt{7})(x + i\sqrt{7})$ the factored form of $f(x) = x^4 + 4x^3 + 7x^2 + 28x$?

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17. Is $g(x) = (x + 1)(x + 2)(4x + 7)$ the factored form of $f(x) = 4x^3 - 11x^2 - x + 14$?

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18. Is $g(x) = (x - 1)(x + 1)(x - 3i)(x + 3i)$ the factored form of $f(x) = x^4 + 8x^2 - 9$?

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Use the Factor Theorem to determine the unknown coefficient so that the given linear expression is a factor of the function.

19. Determine a if $x + 3$ is a factor of $f(x) = x^3 + 9x^2 + ax + 15$.

If $x + 3$ is a factor, then by the Factor Theorem $f(-3) = 0$.

$$f(-3) = (-3)^3 + 9(-3)^2 + a(-3) + 15$$

$$= -27 + 81 - 3a + 15$$

$$= 69 - 3a$$

By the Transitive Property, $69 - 3a = 0$.

$$69 - 3a = 0$$

$$69 = 3a$$

$$a = 23$$

20. Determine a if $x - 4$ is a factor of $f(x) = x^3 + ax^2 - 20x - 48$.

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21. Determine a if $x - 1$ is a factor of $f(x) = ax^3 - 10x^2 - 13x + 20$.

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22. Determine a if $x - 7$ is a factor of $f(x) = x^4 - 4x^3 + ax^2 - 8x - 42$.

23. Determine a if $x + 2$ is a factor of $f(x) = x^3 - x^2 + ax - 36$.

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24. Determine a if $x - 8$ is a factor of $f(x) = x^4 + ax^3 - 5x^2 - 21x - 24$.